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ABSTRACT

This document focuses on evidence from problem solving case studies which indicate that analogy, extreme case analogies, and physical intuition can play an important role as forms of nonformal reasoning in scientific thinking. Two examples of nonformal reasoning are examined in greater detail from 10 case studies of "expert" problem solving. Advanced graduate students and professors in technical fields were asked to think aloud while solving a physics problem. Spontaneously generated analogies were observed to play a role in the solutions, and several subprocesses involved in analogical reasoning were also identified. The use of extreme cases and beliefs based on physical intuition were observed, as well as imagistic prediction reports where the problem solver refers to imagining or picturing an event mentally as he/she makes a prediction. The findings suggest that it may be possible to develop theoretical models for certain patterns of nonformal scientific reasoning. A table and six figures are included. (TW)

NONFORMAL REASONING IN SCIENCE: THE USE OF ANALOGIES AND EXTREME CASES

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Abstract: Examples of nonformal reasoning are examined in case studies of expert problem solving. Advanced graduate students and professors in technical fields were asked to think aloud while solving a physics problem. Spontaneously generated analogies were observed to play a role in the solutions, and several subprocesses involved in analogical reasoning were identified. The use of extreme cases and beliefs based on physical intuition were also observed, as well as imagistic prediction reports where the problem solver refers to imagining or picturing an event mentally as he makes a prediction. The findings suggest that it will be possible to develop theoretical models for certain patterns of nonformal scientific reasoning.

Paper presented at the Conference on "Informal Reasoning and Education" sponsored by OERI, University of Pittsburgh, March, 1987.

NONFORMAL REASONING IN SCIENCE: THE USE OF ANALOGIES AND EXTREME CASES

Introduction

This chapter focuses on evidence from problem solving case studies indicating that analogy, extreme case analogies, and physical intuition can play an important role as forms of nonformal reasoning in scientific thinking. Although some may consider these to be more "casual" methods than deductive reasoning, one of my purposes is to show that they can be used very seriously in a rather formal context-- in this case the context of doing one's best to think about a physics problem.

There are a number of accounts of the role of different types of nonformal thinking in scientific discovery, such as Koestler (1964). These reports are often based on retrospective recounting of the discovery by the scientist. Although they are certainly of value, one limitation of these studies stems from the difficulty of recalling one's exact train of thought from hindsight. Especially when the train of thought leads to a significant conceptual change, it can be difficult to recover a previous state of mind and remember exactly what one's thinking was like before the change. Often multiple sources of ideas contribute to an eventual synthesis and it can be difficult to recall the exact order in the train of ideas. Therefore, it is interesting to ask whether more direct evidence for the role of nonformal thinking in science can be gathered in some way.

The evidence collected for this study comes from videotaped interviews in which scientifically trained subjects were asked to think aloud as they solved problems. Among the few existing psychological studies of analogy, most have focused on provoked analogies, where at least part of the analogy is presented to the subject for completion. This chapter, however, describes research on spontaneous analogies where the subject [illegible] analogy. These occur when a subject, in thinking about problem situation A, shifts, without being prompted, to consider a situation B which differs in some significant way from A, and intends to apply findings from B to A. In successful solutions by analogy the two contexts being compared are often perceptually different but they are always seen to be functionally or structurally similar in some way. Such solutions can sometimes radically restructure the subject's understanding of the problem situation and are most useful in unfamiliar problems where the subject is not able to apply a familiar principle to the problem in a direct manner.

In describing the activities of scientists, philosophers of science have tended to separate the "context of discovery" (hypothesis generation) from the "context of demonstration" (hypothesis testing). The process of hypothesis generation is considered to be much less well understood than the process of hypothesis testing. However, some authors have suggested that reasoning by analogy may play an important role in hypothesis generation [illegible] Campbell [1957], Black [1979], Hesse [1966], Oppenheimer [1956], Nagel [1961], Knorr [1980], and Darden [1983]. Although the problem reported in this study is not a problem on the frontier of science, in each case the subjects were giving a scientific explanation of a phenomenon they were unfamiliar with, i.e., a problem on the frontier of their own personal knowledge. Thus, it is plausible that the thought processes analyzed in this chapter will share some characteristics with thought processes used in scientific research.

Source of Data

One would expect that the use of spontaneous [illegible] might [illegible] a relatively uncommon creative act, since it involves br[illegible] in the assumptions built up in considering the original [illegible], although spontaneous analogies are a more natural [illegible] study than provoked anal[illegible], they are difficult to [illegible]. However, by using unfamiliar problems with scientifically trained subjects, a number of such cases have been documented. Subjects in [illegible] solve the "Spring Problem," shown in Figure 1.

The correct answer to the spring problem is that the wide spring will stretch farther. This seems to correspond to most people's initial intuition about the problem. However, giving a careful justification for this answer is a much more difficult task.

In the problem solving interviews subjects [illegible] the purpose of the interview was to study problem solving methods [illegible] asked to think aloud as much as possible during their [illegible] advanced doctoral students or professors in technical fields. [illegible] problem solver" in this paper I mean a person who [illegible] problem solver in a technical field.

Subjects were given instructions to solve [illegible] "in any way that you can", and were asked to give a rough estimate [illegible] their answer. Probing by the interviewer was kept to [illegible] consisting of a reminder to keep talking. Occasionally [illegible] for clarification of an ambiguous report. All sessions [illegible] in this chapter were videotaped.

Observations from Transcripts

The solutions collected were up to 90 minutes long, and there were a number of different ways that analogies were [illegible] different solutions. The main purpose of this chapter is to present [illegible] examples of the phenomena and to develop constructs for describing and classifying the reasoning patterns observed. I will attempt to provide a close-up view of nonformal thinking in science by concentrating on examples from the protocols of two subjects solving the spring problem.

Analogy Generation

We first examine [illegible] from the solution of [illegible] physicist referred to here as S1. In the following section he generates the "hairpin" analogy shown in Figure 2

> S1: ...The equivalent problem that might have the same answer is-- suppose I gave you the problem in a way instead of being a coiled spring, it's a long U spring like that, just like a hairpin. (Draws Fig. 2) And now I hang a weight on the hairpin, and see how far it bends down. Now I make the hairpin twice as long with the same wire and see how far it bends down. Now that goes with the cube. That's the deflection as the length of the cantilever beam. Heh, heh - and maybe it comes out that way with the spring. So my-- I would bet about, about 2 to 1, I would bet that the answer to this [the original spring problem] is that it [the wider spring] goes down 8 times as far.

Here the subject generates an analogous case in the form of a U-shaped wire or "hairpin". To dispel doubt that analogies are used only by those who lack more formal reasoning methods we note that this subject is a Nobel laureate in physics. He is able to make a confident prediction about the behavior of this analogous case and proposes that he may be able to transfer this prediction to the original problem concerning the spring. Note however that his confidence in this conclusion is not 100% - in his words warranting a bet with onl 2 to 1" odds. Unlike the process of deductive reasoning from given assumptions, reasoning by analogy from given assumptions is an approach that cannot be done with certainty. Apparently it can be done with relativel high or low confidence however, as will be discussed. (Here the subject attempted to make a quantitative prediction as well as a qualitative prediction about which spring would stretch more. In the remainder of this chapter we will be concerned only with the qualitative aspects of the problem.)

The 10 subjects generated thirty eight analogies altogether. An analogy was classified as significant if it appeared to be part of a serious attempt to generate or evaluate a solution, and as non-significant if it was simply mentioned as an aside or commentary. Thirty-one of the analogies were significant accord g to this criterion. Eight of the ten subjects generated at least one analogy, and seven of the ten generated at least one significant analogy. Thus a large number of analogies were observed.

In what follows it will be useful to distinguish between two parts of an analogy, the analogous case and the analogy relation. The analogous case in the above example is the hairpin experiment itself, and the analogy relation is the relationship being proposed by the subject of a partial equivalence between the original case involving springs and the analogous case involving the hairpin. The subject appears to have high confidence in his understanding of the analogous case, but only moderate confidence in the validity of the analogy relation. In other cases subjects have been observed to reject the validity of an analogy relation, that is, they would state that the analogous case was not similar enough to the original problem to draw any conclusions from it.

Subprocesses Used in Analogical Reasoning

From observations of this kind the general hypothesis was formulated that the following subprocesses are fundamental in making an inference by analogy (Clement 1982, to appear (b)):

(P1) Generating the Analogy. A conception of a situation B that is potentially analogous to A is accessed in memory or constructed. A tentative analogy relation is set up between A and B.

(P2) Confirming the Analogy Relation. The validity of the analogy relation between A and B is examined critically and is confirmed at a high level of confidence.

(P3) Understanding the Analogous Case. The subject examines and, if necessary, develops his or her understanding of the analogous case B, and the behavior of B becomes well-understood, or at least predictable.

(P4) Transferring Findings. The subject transfers conclusions or methods from B back to A.

Table 1

This hypothesis is consistent with the further observation that many solutions by analogy are proposed tentatively, and processes (P2) and (P3) especially can be quite time consuming. (When it is not clear from the context, the word "analogy" alone refers in this chapter to the analogous case and the analogy relation taken together.) Observations also indicate that the last three processes can be initiated in any order, and that subjects can go back and forth between them several times while gradually completing each subprocess. This suggests that the subjects do not use a simple, well-ordered procedure for controlling their solution processes at this level.

Analogies and Extreme Cases from a Second Subject

We will next examine the solution of a subject, S3, who is an advanced doctoral candidate in computer science, and [illegible] electrical engineer. The actual protocols for difficult problems [illegible], therefore I present verbatim segments of protocols [illegible] Immediately after reading the problem, S3 proceeds as follows:

008 S3: ...Umm, well right off the bat I have no idea. Umm, and my first thought is that the length of the [illegible] spring being greater (traces circles in air with finger [illegible] downward) and the strength of the metal being the same means that there's going to be kind of more leverage for [illegible] [in the wider spring].

009 S3. And that therefore it's going to hang [illegible]. And that's pretty much strictly an intuition based on my familiarity with metal and with working with metal... Let me just think through that..

010 S3: (Draws horizontal rods in fig 3) ..And my intuition about that is that if you took the same wire that [illegible] on the left here [short horizontal rod] and doubled [illegible] and hang some weight on it, that the same [illegible] weight on it, would bend considerably further.

019 S3: It would seem that that means that um, that back in the original problem, the spring in picture 2 [the wider spring] is going to hang farther; it's going to [illegible] more.

021 S3: ...and I have a confidence of [illegible]

022 S3: ...I have a great deal of [illegible] displacement of the long rod] is greater [illegible] of the short rod] in any case. I would [illegible] confidence.

Further evidence for [illegible] major episodes appearing in this first section are

(1) S3 first describes thinking about an "intuition" which predicts that the larger spring will stretch farther;

(2) line 10 indicates that he has spontaneously [illegible] an analogy when he draws the picture of an analogous problem involving a [illegible] rod instead

of stretched spring. He decides, again on the basis of "intuition", that the long rod would bend more than the short rod and is able to state a 100% level of confidence for this. This indicates that he has completed processes P1 and P3 in Table 1 (generating and comprehending the [illegible]).

(3) He gives evidence for completing step P4 (transferring findings) in line 19, where he says that his analogy indicates that the larger spring in the original problem will stretch further.

However, he is still not 100% certain about his answer to the original problem. A plausible explanation for this lack of confidence is that he is not fully satisfied with requirement P2 above (evaluating the analogy relation between A and B).

This transcript and others indicate that processes P1 through [illegible] above can indeed take place separately. S2 has apparently completed processes P1, P3, and P4 so far. Note that as described here, [illegible] findings, can take place before steps P2 and P3 are [illegible]. In other words, a tentative prediction about the original [illegible] made before the analogy relation has been confirmed or before the [illegible] case is fully understood. This is another sense in which [illegible] can involve a conjecture.

In order to begin the task of modeling the [illegible] cognitive processes responsible for this type of analogical reasoning, it will be useful to use the notation in Figure 4 showing the four major subprocesses. In this notation, dotted squares and solid squares represent [illegible] understood and well understood conceptions, respectively. Dotted and solid lines between squares represent unconfirmed and confirmed analogy relations between conceptions, respectively. Again, the order in which [illegible], P3 and P4 are initiated may vary

A diagram showing the status of the analogy at the end of the above protocol section is shown in Figure 5. A poorly understood conception of the spring is linked by analogy to a well understood conception of the rod. The dotted line in Figure 5 indicates that the analogy relation has not been confirmed. That is, even though the subject is sure that he understands how the bending rod situation works, he is still unsure that that situation is a good analogy for a spring, i.e., that the situation can be considered to be equivalent to a spring and can be used to predict the behavior of the spring. Thus, we refer to a <u>tentative or unconfirmed analogy relation</u> at this point.

<u>Extreme cases</u> Subjects that spent less than 1 hour on this problem without reaching a complete answer or 100% confidence level were asked to spend more time considering the problem, in order to push their confidence level up higher. In this case this leads the subject to take a new approach.

030 I: Ok, let me push you a little on this [illegible] any way you can increase your confidence in your prediction?

049 S3: Ok. Good. Um, well the way to increase my confidence would be to examine the contrary hypothesis that the notion that um...the stretch is the same or [illegible] less.

050 S3: Here's the thought experiment that I [illegible] perform.

051 S3: The way to really eke out my intuition [illegible] the behavior of the material is at all linear, would [illegible] take the coiled spring in 1 down to... Make the [illegible] extremely tightly coiled [even [illegible]] It's still only to 5 turns.

052 S3: It's very clearly in the limit.. It's almost ... no distance from side to side of the spring. [illegible] that case it can't stretch very far... There isn't material to [illegible] from to contribute to a stretch. So um, my intuition that my answer's correct has just jumped up to 85 or 90% as I examined that in the one extreme... As you make that smaller (brings palms of hands close together) it's going to stretch less

The above excerpt provides an example of extreme case analysis, where the subject minimizes or maximizes an aspect of the problem to create a special case that may be easier to solve. In this case, considering an extremely narrow spring allows the subject to make a more confident prediction based on what he calls physical intuition. The subject continues by generating and attempting to use a second extreme case:

053 S3: And just to really push that thinking, um, all the way through in B, if we made those coils immensely long, (moves hands apart) say miles wide.
Um, well there's a problem with thinking about that which is, that..

054 S3: If you made the spring very big then um, then the mass of the spring starts coming into effect and I have trouble separating my intuition about thinking about a huge spring that [where] the mass isn't a problem...

055 S3: So going in that direction doesn't really help me too much...But I think it was very constructive to go back and look at what happens if we go down in the diameter of the spring...

056 S3: ...My confidence is now much higher um, er, 85 95% (stares at drawing). Even more [illegible] Even more [illegible]

057 I: Did anything new happen to get the 'even more' or...?

058 S3: Just I was thinking about, I was just running, let my intuition about that really taking the diameter of the spring to zero and the limit. In which case the stretch goes to zero.

059 I: How do you feel when you're "running that intuition"?

062 S3: Um, it's just I have a; I mean the process of taking the diameter of the spring to zero is a straight wire, which wouldn't stretch...

066 S3: So it's good ..pushing the parameters of the problem to extremes as a way of-uh, getting clearer intuitions about the behavior of a system.

In the section above, S3 mentions thinking about his first thought experiment over again, involving an extreme case of a very narrow spring, and takes it even further by letting the width actually go to zero, in which case the spring becomes a straight wire. The extreme case here seems to increase his confidence even more.

Physical intuition and imagery reports. Subject S3 indicates that he relies on some sort of physical intuition to make predictions by line 10. The subject refers to his prediction that a long wire will bend more than a short wire as an "intuition." This suggests that he is using some knowledge based on previous concrete experiences with manipulating metal. Here I will use the term "physical intuition schema" to refer to an internal knowledge structure of this type that is constructed largely on the basis of personal experience with the physical world rather than academic knowledge or hearsay.

The subject also reports thinking about a "picture" in line 62 in the above section. This is an example of an imagery report where the subject refers to imagining, picturing, hearing, or "feeling what it's like to manipulate" a situation. Of course subjects may actually experience imaging much more often than they make imagery reports. In the example in line 62 above the subject also makes a prediction. In this case we also call it an imagistic prediction report where the subject produces an imagery report and relatively simultaneously states a prediction or conclusion. (The interviewer was careful not to be the first to introduce suggestive terms such as "image", "picture", or "analogy" in the interviews.)

I can now state several further hypotheses that are suggested by the above observations.

First, the co-occurrence of imagistic prediction reports and intuition reports in line 62 above and 85 below, suggests that the process of using physical intuitions here involves imagery.

Second, a major function of the extreme cases appears to be to enable the use of a physical intuition schema in making an imagistic prediction with high confidence.

Third, the subject experiences difficulty with using a previous extreme case in the first part of the above section. He has difficulty thinking about a huge massless spring in attempting to construct an extreme case in the opposite direction. Even though he recognizes that he needs to separate out the variable of mass of the spring from the problem, he apparently has difficulty voluntarily carrying out a thought experiment which includes that constraint. This suggests that there are certain limitations associated with the implementation of thought experiments that involve imagery and physical intuition.

The interviewer then pushes the subject once more.

067 I: Is there any way you could increase your confidence even more...?

085 S3: ...I guess er, my tendency is to think about a big spring. Push the...diameter up and picture in my mind a really big spring with that weight hanging from it. And uh, it's just really obvious that it's gonna hang further ..

086 I. What are you thinking about there?

087 S3: I have a picture in mind...

089 S3: I have a picture in mind. I see many, uh; I flashed on the image of the Foucault Pendulum at the Smithsonian Institute which had nothing to do with this except it's a big physical system (S laughing)...Um, so I'm wishing that they had a big spring hung out there, so that I could have an even clearer picture.

Line 85 contains another example of an imagistic prediction report. In this second attempt the subject seems to be more successful in thinking about the behavior of a very large spring. The subsequent Foucault pendulum idea, referring to another system involving a long wire and a weight, appears to derive from an associative analogy generation process, but this idea is discounted immediately, apparently due to a lack of structural similarity between the two cases.

Summary of S3's protocol. Subject S3 gives evidence of using a number of different approaches. He makes use of an *analogy* to a simpler situation involving bending rods of different lengths. He uses *physical intuition* beliefs to make predictions about such simpler situations. When a prediction is accompanied by an imagery report, we call it an *imagistic prediction report*. Finally, he uses *extreme cases*, a very narrow spring and a very wide spring, to further support his initial answer. In both cases, the apparent function of the extreme cases is to facilitate the confident application of a physical intuition belief to the problem.

The use of multiple approaches to the problem appears to allow the subject to increase his confidence in his solution. S1, the first subject examined in this chapter, also went beyond the use of an analogy in his first approach to the problem by using more analytic methods in order to confirm his predictions generated by the hairpin analogy.

A more subtle understanding of exactly how the spring deforms arose from a third subject's generation of the analogy of a square shaped spring coil. This allowed him to discover that the spring wire is twisting as it stretches. In the square shaped coil one can envision one of the sides of the square acting like a wrench to twist the next side-- and so on down the spring (Clement, 1981, to appear (a)). (The square coil can also be used to predict the result that the stretch of the spring depends on the cube of the coil's diameter.) Thus, a variety of analogous cases were observed for this problem.

Discussion

Nonformal Knowledge vs. Nonformal Reasoning

In addition to nonformal reasoning processes such as the use of analogies, examples have also been presented of imagery reports and imagistic prediction reports which indicate the use of physical intuition knowledge. It is helpful to make a distinction between nonformal knowledge and nonformal reasoning. In the case of S3 the conclusions he reaches seem to be grounded at the most basic level on physical intuition schemas constructed from prior experiences with physical objects (e.g. long objects are easier to bend than short objects) rather than formal knowledge assumptions. Thus he uses a kind of nonformal knowledge. Analogical reasoning and extreme case reasoning allow him to transfer this knowledge, with some degree of confidence to the given problem situation. These two types of nonformal reasoning, then, allow him to apply or transfer his nonformal knowledge in the form of physical intuitions to the problem.

Flexibility and Uncertainty

The flexibility exhibited in scientific thinking that involves extreme cases and analogies is impressive. Such flexible methods play an important role in the hypothesis generation process in science[1]. Analogy generation is a creative and divergent process, since the subject must somehow break away from the normal assumptions implied by the problem and shift his attention over to a significantly different but related problem. This is difficult for some people to do, probably because of the difficulty involved in breaking out of the psychological "set" of assumptions built up in considering the original problem.

It is also somewhat risky, because there is no guarantee ahead of time that the results will be found to pay off-- one does not have the security of perceived certainty that is experienced in deductive reasoning. However, at the end of the protocol, subject S3 is "95% confident" in his (correct) answer to the prediction about the behavior of the spring. He achieves this level without using formal methods. Presumably, the fact that he has arrived at the same prediction in three different ways has played an important role in boosting his confidence. Even though individual nonformal reasoning methods involve a degree of uncertainty, the convergence of several methods on the same result can raise confidence levels to a high point [2]

Summary

In this very brief chapter, examples have been presented which provide evidence for the presence of the following types of spontaneous nonformal reasoning phenomena:

(1) the use of analogous cases;

(2) the use of extreme cases;

(3) the presence of four subprocesses involved in using analogical reasoning;

- (A) generating the analogy
- (B) confirming the analogy relation
- (C) understanding the analogous case
- (D) transferring findings;

(4) the presence of various levels of confidence in different beliefs and reasoning steps;

(5) the presence of imagery reports and imagistic prediction reports which indicate the use of nonformal knowledge in the form of physical intuition schemes.

The above phenomena have also been observed in mathematical thinking (Polya, 195., Clement, 1983). The fact that we can now collect and describe such phenomena suggests that it will be possible to develop and evaluate models and theories for certain patterns of nonformal scientific reasoning.

Notes

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1. See, for example, Hesse (1966) and Oppenheimer (1955). Clement (to appear ( )) discusses evidence for the generation of hypothetical models and analogies in thinking about protocols.

2. See Clement (1986) for a discussion of a number of methods experts can use to increase their confidence in the validity of an analogy.

REFERENCES

Black, M. (1979). More about metaphor. In A. Ortony (Ed.) Metaphor and thought. Cambridge, England: Cambridge University Press.

Campbell, Norman R. (1957). Foundations of science. New York: Dover.

Clement, J. (1981). Analogy generation in scientific problem solving. Proceedings of the Third Annual Conference of the Cognitive Science Society. Berkeley, Ca.

Clement, J. (1982). Analogical reasoning patterns in expert problem solving. Proceedings of the Fourth Annual Meeting of the Cognitive Science Society. Ann Arbor, MI.

Clement, J. (1986). Methods for evaluating the validity of hypothesized analogies. Proceedings of the Eighth Annual Conference of the Cognitive Science Society, Hillsdale, NJ: Lawrence Erlbaum Associates.

Clement, J. (to appear (a)). Creativity in Science. In Glover, J., Ronning, R., & Reynolds, C., Handbook of Creativity. New York, NY: Plenum.

Clement, J. (to appear (b)). Observed methods for generating analogies in scientific problem solving. Cognitive Science.

Darden, L. (1983). Artificial intelligence and philosophy of science reasoning by analogy in theory construction. In Philosophy of Science Association, 2, 147 - 165.

Hesse, M. (1966). Models and analogies in science. South Bend, IN: University of Notre Dame Press.

Knorr, K. (1980). The manufacture of knowledge. Cambridge, England: Oxford University Press.

Koestler, (1964). The act of creation.

Nagel, E. (1961). The structure of science: Problems in the logic of scientific explanation. New York: Harcourt, Brace & World.

Oppenheimer, R. (1956). Analogy in science. The American Psychologist, 11(3), 127 - 135.

Polya, G. C. (1954). Mathematics and plausible reasoning, vol. 1: Induction and analogy in mathematics. Trenton, NJ: Princeton University Press, 155.

(P1) Generating the Analogy. A conception of a situation B that is potentially analogous to A is accessed in memory or constructed. A tentative analogy relation is set up between A and B.

(P2) Establishing Confidence in the Analogy Relation. The validity of the analogy relation between A and B is examined critically and is confirmed at a high level of confidence.

(P3) Understanding the Analogous Case. The subject examines and, if necessary, develops his or her understanding of the analogous case B, and the behavior of B becomes well-understood, or at least predictable.

(P4) Applying Findings. The subject applies conclusions or methods gained from B back to A.

Table 1

SPRING PROBLEM

A WEIGHT IS HUNG ON A SPRING. THE ORIGINAL SPRING IS REPLACED WITH A SPRING:

--MADE OF THE SAME KIND OF WIRE,
--WITH THE SAME NUMBER OF COILS
--BUT WITH COILS THAT ARE TWICE AS WIDE IN DIAMETER.

WILL THE SPRING STRETCH FROM ITS NATURAL LENGTH, MORE, LESS, OR THE SAME AMOUNT UNDER THE SAME WEIGHT? (ASSUME THE MASS OF THE SPRING IS NEGLIGIBLE COMPARED TO THE MASS OF THE WEIGHT.)

WHY DO YOU THINK SO?

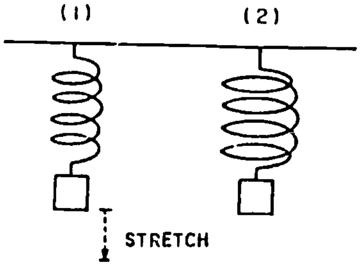

Figure 1

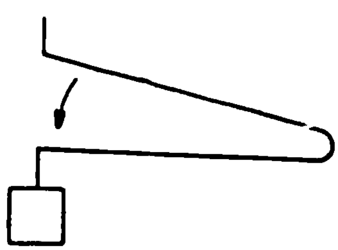

Figure 2

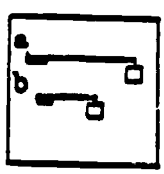

FIGURE 3

MAJOR STEPS IN SUCCESSFUL USE OF A SPONTANEOUS ANALOGY

GIVEN POORLY UNDERSTOOD PROBLEM SITUATION A

P1) GENERATE TENTATIVE ANALOGOUS CASE B

P2) CONFIRM THE VALIDITY OF THE ANALOGY RELATION

P3) UNDERSTAND CASE B

P4) TRANSFER PREDICTION FROM B TO A

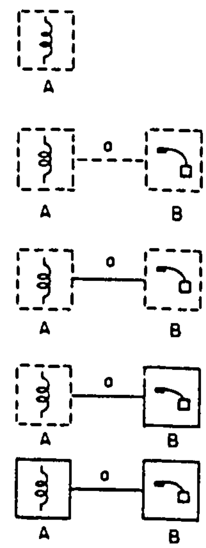

FIGURE 4

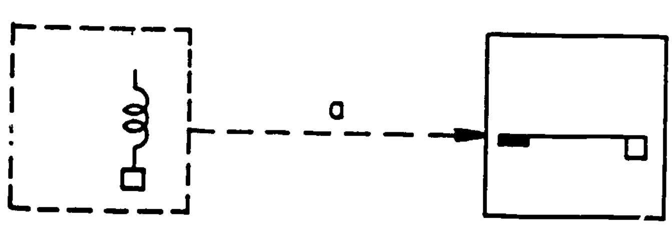

FIGURE 5

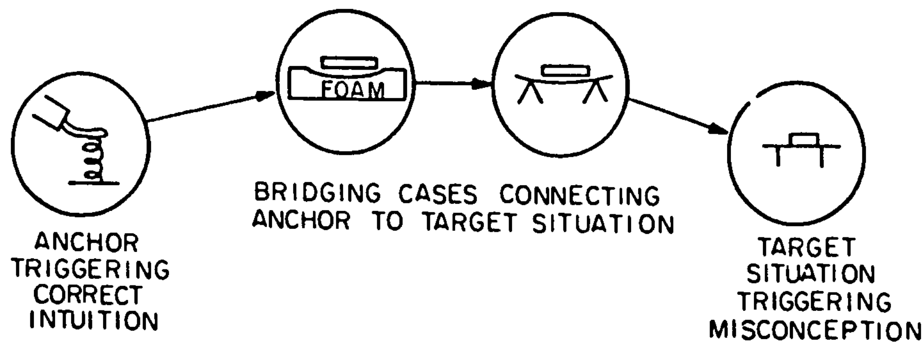
FOAM
BRIDGING CASES CONNECTING ANCHOR TO TARGET SITUATION
ANCHOR TRIGGERING CORRECT INTUITION
TARGET SITUATION TRIGGERING MISCONCEPTION

Figure 6